



# Math Virtual Learning

## Precalculus with Trigonometry

Students will find the exact value of angle measures using Sum and Difference Identities for Sine and Cosine

**April 28, 2020**



# Precalculus with Trigonometry

## Lesson: April 28th, 2020

### **Objective/Learning Target:**

Students will find the exact value of angle measures using Sum and Difference Identities for Sine and Cosine.

Today's lesson will introduce you to the Sum and Difference Identities for sine and cosine (we'll do tangent tomorrow) and how you use the identities to find the exact value for angle measures not on the unit circle.

**Watch Video:**

[Using Sum and Difference Identities](#)

## Sum and Difference Identities

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Example 1: Find the exact value of  $\cos(75^\circ)$

$$\begin{aligned}\cos 75^\circ &= \cos (45^\circ + 30^\circ) \\ &= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \\ &= \frac{\sqrt{2}}{2} \left( \frac{\sqrt{3}}{2} \right) - \frac{\sqrt{2}}{2} \left( \frac{1}{2} \right) \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

Substitute  $45^\circ + 30^\circ$  for  $75^\circ$ .

Sum formula for cosine

Evaluate.

Simplify.

Example 2: Find the exact value of  $\cos \frac{11\pi}{12}$

$$\begin{aligned}\cos \frac{11\pi}{12} &= \cos \left( \frac{2\pi}{3} + \frac{\pi}{4} \right) \\ &= \cos \frac{2\pi}{3} \cos \frac{\pi}{4} - \sin \frac{2\pi}{3} \sin \frac{\pi}{4} \\ &= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= -\frac{\sqrt{2} + \sqrt{6}}{4}\end{aligned}$$

Alternatively, you can convert radian to degree measure, and then evaluate

$$\cos \frac{11\pi}{12} = \cos(165^\circ) = \cos(120^\circ + 45^\circ)$$

# Practice

Use the Sum and Difference Identities to find the exact value of the following:

1.  $\sin(105^\circ)$

2.  $\cos(195^\circ)$

3.  $\sin\left(\frac{7\pi}{12}\right)$

4.  $\cos\left(\frac{5\pi}{12}\right)$

# Practice - **ANSWERS**

$$\sin(105^\circ) = \sin(60^\circ + 45^\circ) = \sin(60^\circ)\cos(45^\circ) + \cos(60^\circ)\sin(45^\circ) = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\cos(195^\circ) = \cos(150^\circ + 45^\circ) = \cos(150^\circ)\cos(45^\circ) - \sin(150^\circ)\sin(45^\circ) = \frac{-(\sqrt{2} + \sqrt{6})}{4}$$

$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

# Additional Practice and Resources:

Additional Resource Videos:

[Proof of Sum and Difference Formula for Sine](#)

[Finding exact value of cosine with radian measure](#)

Additional Practice:

[Sum and Difference practice - Kuta](#)

*Try problems 1 - 10*